A Non-Linear Fluid-solid Coupling Mechanical Model Study for Paleokarst Collapse Breccia Pipes Under Erosion Effect

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ABSTRACT

In this research the seepage characteristics of Paleokarst collapse breccia pipes under particles erosion effect, and their water inrush mechanism were studied. In this paper, based on the seepage theory of pores media and the nonlinear mechanics theory, we deduced the transport equation of particles in Paleokarst collapse breccia pipes, obtained the seepage field equation for Paleokarst collapse breccia pipes, and investigated the porosity evolution equation under the effect of particles transport, building a nonlinear fluid-solid coupling model for Paleokarst collapse breccia pipes. Furthermore, we took the relationship between fluid and particle velocities as well as the effect of particle concentration on fluid property into account, and assumed the porosity in Paleokarst collapse breccia pipes obey Weibull distribution. Finally, we lead the model equations into the COMSOL Multiphysics to solve, obtaining the parameters including porosity, seepage velocity, particle concentration, water inflow evolution law as the time. The research results indicate that: (1) particles in Paleokarst collapse breccia pipes will be eroded and transport under the effect of fluid movement as the time, the concentration of particles behaved rapidly increased and then sharply decreased, and the porosity and seepage velocity grew quickly until reached the maximum value; (2) the seepage capacity for Paleokarst collapse breccia pipes initially grows slowly, while seepage velocity increases at an increasing rate with the growth and connectivity of porosity; (3) the porosity evolution under erosion effect in Paleokarst collapse breccia pipe is an important reason for Paleokarst collapse breccia pipe water inrush.

KEYWORDS: Paleokarst collapse breccia pipe; fluid-solid coupling model; erosion effect; numerical simulation.
INTRODUCTION

As a special vertical development structure in rock strata (Figure 1), Paleokarst collapse breccia pipes universally distribute in more than 20 coal fields in North China including Shanxi Platus, Taihang montain, Lvliang mountain as well as Shandong, Henan, Jiangsu and Shanxi province. Among the assumption on the formation of Paleokarst collapse breccia pipes, one representative explanation is: due to transport of groundwater, Carbonate rock is gradually eroded and Karst caves formed gradually, then the upper strata collapsed due to the long-period effect of geological structural stress and gravity, and formed Paleokarst collapse breccia pipes finally. Because the Paleokarst collapse breccia pipe connect with Ordovician confined aquifer which are characterized by high water pressure and rich water supply, water inrush accidents often happen during working face go through Paleokarst collapse breccia pipes, causing huge economical loss and serious people’s death. Therefore, water-conducted Paleokarst collapse breccia pipes become hidden trouble which threaten and influence the safety mining in many mining areas in China. In order to explore the water inrush mechanism of water-conducted Paleokarst collapse breccia pipe, many researchers in China has done many useful work. For instance, in our previous work[1], we established a mechanical model for predicting concealed Paleokarst collapse breccia pipes water inrush, some researchers in China confirmed that natural hydraulic fracturing effect if an important reason for water inrush of strong water conductive Paleokarst collapse breccia pipe through analysis on water inrush accidents and simulation experiment[2]; and some researchers in China studied the water inrush process of Paleokarst collapse breccia pipe by using numerical simulation software, and discussed the water inrush mechanism[3]. However, these investigations mainly carried out from respect of structure failure, only few researchers in China investigated this problem from perspective of fluid-solid coupling. In this paper, we consider the water inrush accidents caused by Paleokarst collapse breccia pipe as the reason of erosion, which can result in the porosity and permeability change, and induced water inrush accidents. Due to lack of research in this field in China, we refer some papers on sand erosion, which can provide useful reference for our study. For example, the problem of aqueous solutions that flow through a porous rock and react with its mineral components were studied, and introduce a equation on porosity change rate, established a equation on porosity change and fluid advection and diffusion effect [4]; M.A. Habib investigated the erosion rate correlations of a pipe protruded in an abrupt pipe contraction problem and presented a numerical investigation of the erosion of a pipe protruded in a sudden contraction[5]. Other papers[6-9] also provide useful reference for our investigation. Based on these studies, we carried out our research from the perspective of fluid-solid coupling effect, which have significant meaning for revealing water inrush mechanism and preventing water inrush accidents of Paleokarst collapse breccia pipes in China.

Figure 1: Paleokarst collapse breccia pipes in rock strata
ESTABLISHMENT OF NON-LINEAR MECHANICAL MODEL

Basic Assumptions

1) Paleokarst collapse breccia pipes can be considered as pores media;

2) Transport of fluid and particles in Paleokarst collapse breccia pipes obey Darcy’s law;

3) Concentration of fluidized solid particles in fluid is relatively small so impact effect between particles can be neglected.

Definition

Based on the above assumption, we can take a representative element volume(REV), as shown in Figure 2, which consists of the following three constituents: solid(s), fluid(s) and fluidized solid particles, with volumes \( V_s, V_f, V_{fs} \) and masses \( M_s, M_f, M_{fs} \) respectively.

Therefore the void volume in the REV is \( V_v = V_f + V_{fs} \), and the porosity of REV can be expressed as:

\[
\phi = \frac{V_v}{V} = \frac{V_f + V_{fs}}{V} \quad (1)
\]

Concentration of fluidized solid particles in porosity is:

\[
C = \frac{V_{fs}}{V_v} = \frac{V_{fs}}{V_f + V_{fs}}, \quad \rho_{fs} = \frac{M_{fs}}{V_v} = C \rho_s \quad (2)
\]

Figure 2: A representative element volume for Paleokarst collapse breccia pipe
Mass Balance Equations

We assume the coordinate of REV is \( P(x, y, z) \), with volume \( V = dx dy dz \), and it is small but can contain all information.

The transport of fluidized solid particles is caused by the coaction of advection and diffusion. For advection effect along \( z \) direction, the lower surface area is \( dx dy \), with a velocity of \( q_{pz} \), therefore the fluidized solid particles accumulated in REV in \( z \) direction during unit time can be expressed as:

\[
\frac{\partial (\rho f q_{pz})}{\partial z} dx dy dz
\]

(3)

And the fluidized solid particles accumulated in REV in \( z \) direction during unit time under the effect of diffusion can be expressed as:

\[-\frac{\partial}{\partial z} (\phi D \frac{\partial \rho f}{\partial z}) dx dy dz\]

(4)

Therefore the total accumulation of fluidized solid particles in the REV caused by the coaction of advection and diffusion in \( z \) direction during unit time can be expressed as:

\[\frac{\partial}{\partial z} (\rho f q_{pz} - \phi D \frac{\partial \rho f}{\partial z}) dx dy dz\]

(5)

By the same way we can obtain the mass accumulation in \( x \) and \( y \) direction during unit time.

Thus for the REV, the total mass accumulation of fluidized solid particles in the REV is:

\[\frac{\partial}{\partial x} (\rho f q_{px} - \phi D \frac{\partial \rho f}{\partial z}) dx dy dz + \frac{\partial}{\partial y} (\rho f q_{py} - \phi D \frac{\partial \rho f}{\partial z}) dx dy dz + \frac{\partial}{\partial z} (\rho f q_{pz} - \phi D \frac{\partial \rho f}{\partial z}) dx dy dz\]

(6)

According to mass conservation, the mass accumulation should be equal to the decrease of mass in the REV:

\[-\frac{\partial}{\partial t} (\phi \rho f dx dy dz) + \dot{m} dx dy dz\]

(7)

where \( \dot{m} \) is a mass-generation term, which correspond to the rate of net mass eroded in the REV during unit time, then we can obtain the following expression:

\[-\frac{\partial}{\partial t} (\phi \rho f dx dy dz) + \dot{m} dx dy dz = \frac{\partial}{\partial x} (\rho f q_{px} - \phi D \frac{\partial \rho f}{\partial z}) dx dy dz + \frac{\partial}{\partial y} (\rho f q_{py} - \phi D \frac{\partial \rho f}{\partial z}) dx dy dz + \frac{\partial}{\partial z} (\rho f q_{pz} - \phi D \frac{\partial \rho f}{\partial z}) dx dy dz\]

(8)
We assume the solid particles can not be compressed, and the volume of the REV is constant, we can deduce the following expression with the Eq.2 that is:

$$\frac{\partial (C \phi)}{\partial t} + \nabla \cdot (C \bar{q}_p - \phi D \nabla C) = \frac{m}{\rho_s}$$  \hspace{1cm} (9)

The relationship between porosity change rate and mass-generation term is:

$$\frac{\partial \phi}{\partial t} = \frac{\dot{m}}{\rho_s}$$  \hspace{1cm} (10)

Combining equations (10) and (11) we obtain:

$$\frac{\partial (C \phi)}{\partial t} + \nabla \cdot (C \bar{q}_p - \phi D \nabla C) = \frac{\partial \phi}{\partial t}$$  \hspace{1cm} (11)

For fluidized solid particles, because diffusion effect is relatively smaller than advection effect, we can neglect the diffusion effect, and the (12) results,

$$\frac{\partial (C \phi)}{\partial t} + \nabla \cdot (C \bar{q}_p) = \frac{\partial \phi}{\partial t}$$  \hspace{1cm} (12)

Eq.12 is the particle balance equation.

We can also deduce the balance equation for fluid:

$$\frac{\partial [\phi(1-C)]}{\partial t} + \nabla \cdot [(1-C)\bar{q}] = 0$$  \hspace{1cm} (13)

**Porosity Evolution Equation**

For simplicity and due to lack of experimental data we will study here the governing equations and present results using the monomial erosion model:

$$\frac{\partial \phi}{\partial t} = \lambda \rho_s (1 - \phi) c \bar{q}$$  \hspace{1cm} (14)

Where $\lambda$ is a constant, $\rho_s$ is solid particle density, and $\bar{q}$ is the velocity of mixture, which can be considered as equal to fluid velocity under a small concentration of fluidized solid particles. The expression shows that the change rate of porosity caused by erosion is proportional to the concentration of fluidized solid particles and seepage velocity of mixture.

**Fluid Movement Equation**

The Darcy’s law for fluid movement in porous medium can be expressed as:

$$\bar{q} = -\frac{k}{\eta} (\nabla p + \rho g \nabla z)$$  \hspace{1cm} (15)
In this equation, \( \bar{q} \) is the Darcy velocity (m/s); \( k \) is the permeability of the porous medium (m²); \( \eta \) is the fluid’s dynamic viscosity (Pa·s); \( p \) is the fluid’s pressure (Pa) and \( \rho \) is its density (kg/m³); and \( \nabla z \) is a unit vector in the direction over which \( g \) acts.

**Particle Movement Equation**

Due to fluidized solid particles transport upwardly, their velocity will not be equal to fluid because their density are larger than fluid’s. The velocity of fluidized solid particles under upward transport can be expressed as the difference between fluid velocity and the free settlement velocity of fluidized solid particles in static water, therefore, we have:

\[
\begin{align*}
q_{px} &= \frac{k(\phi)}{\eta} \frac{\partial p}{\partial x} \\
q_{py} &= \frac{k(\phi)}{\eta} \frac{\partial p}{\partial y} \\
q_{pz} &= \frac{k(\phi)}{\eta} \left( \frac{\partial p}{\partial z} + \rho g \right) - \phi \sqrt{\frac{4 d_p (\rho_p - \rho) g}{3 C_D \rho}}
\end{align*}
\]

where \( u_t = \sqrt{\frac{4 d_p (\rho_p - \rho) g}{3 C_D \rho}} \) is the free settlement velocity of fluidized solid particles in static water, \( d_p \) is the diameter of particles, and \( C_D \) is dimensionless drag coefficient, which relate to the Reynolds number \( (Re_p) \), and \( Re_p = \frac{d_p q_p \rho}{\mu} \). J. M Ferreira[10] introduced a drag coefficient which can be suitable within a wide range of Reynolds number \( (0 < Re_p < 1 \times 10^5) \), that is:

\[ C_D = \frac{24}{Re_p} + 0.5 \]

**Auxiliary Equations**

The relationship between seepage velocity for fluid transport in porous medium and porosity can be expressed as:

\[
k(\phi) = k_0 \left( \frac{\phi}{\phi_0} \right)^3 \left( 1 - \frac{\phi}{1 - \phi_0} \right)^2
\]

where, \( \phi_0 \) and \( k_0 \) are the initial porosity and permeability respectively.

The relationship between dynamic viscosity of fluid \( \eta_0 \) and dynamic viscosity \( \eta \) of fluid with particles can be expressed by Einstein formula

\[
\eta(C) = \eta_0 (1 + 2.5C)
\]
To sum up, we have seven equations: (12) ~ (18), and seven unknown parameters: \( \phi, k, p, \eta, C, \tilde{q}, \tilde{q}_p \), therefore the equations can be closed. We can see that this model equations are nonlinear and involve seepage and erosion, which composed the nonlinear fluid-solid coupling mechanical model for Paleokarst collapse breccia pipes.

**Figure 3:** Coupling relationship between model equations

**NUMERICAL SIMULATION**

**Establishment of numerical model**

Based on the nonlinear mechanical model discussed above, we can import it into the coupled software-COMSOL Multiphysics and the variation of various parameters against time could be analyzed.

The numerical model was shown in Figure 4. The width of Paleokarst collapse breccia pipes is 10m and the height is 20m. There is the confined water in the bottom and the water pressure is 2 MPa. The top is the outlet of water where the water pressure is 0. The left and right is the impermeable boundary. The initial water pressure in the model is 2 MPa at the bottom and 0 MPa at the top of model. The initial particle density is \( C_0 = 0.01 \). The other parameters of the model are shown in Table 1.
The distribution of rock heterogeneity could be described by means of Weibull\[11,12\]. The probability density equation in Weibull distribution could be expressed as

\[
f(\phi) = \frac{m}{\phi_0} \left(\frac{\phi}{\phi_0}\right)^{m-1} \exp\left[-\left(\frac{\phi}{\phi_0}\right)^m\right]
\]

(19)

where, \(\phi\) indicates the porosity, \(\phi_0\) denotes average porosity and \(m\) is the uniformity index. The larger \(m\) represents more uniformity. The porosity distribution obtained by numerical generation method is shown in Figure 5(a).

**Table 1:** The values for the main parameters

<table>
<thead>
<tr>
<th>Main parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water dynamic viscosity (\eta_0)</td>
<td>1e-3 (Pa*s)</td>
</tr>
<tr>
<td>Water density (\rho)</td>
<td>1000 (kg/m(^3))</td>
</tr>
<tr>
<td>Initial permeability (k_0)</td>
<td>5e-12 (m(^2))</td>
</tr>
<tr>
<td>Average porosity (\phi_0)</td>
<td>0.15</td>
</tr>
<tr>
<td>Particle diameter (d_p)</td>
<td>1e-5 (m)</td>
</tr>
<tr>
<td>Particle density (\rho_s)</td>
<td>2000 (kg/m(^3))</td>
</tr>
<tr>
<td>Initial concentration (C_0)</td>
<td>0.01</td>
</tr>
<tr>
<td>Initial pressure (p_0)</td>
<td>2 (MPa)</td>
</tr>
<tr>
<td>Erosion coefficient (\lambda)</td>
<td>1e-3 (m(^{-1}))</td>
</tr>
<tr>
<td>The maximum porosity (\phi_{\text{max}})</td>
<td>0.4</td>
</tr>
<tr>
<td>Weibull uniformity index (m)</td>
<td>5</td>
</tr>
</tbody>
</table>
Numerical Simulation Results and Analysis

The porosity distribution in Paleokarst collapse breccia pipes is shown in Fig 5(a) and the porosity distribution after different erosion time shown in Fig 5 (b)–(d). We could learn that the porosity increases with the seepage time. In the initial stage, the porosity in Paleokarst collapse breccia pipes is random and several strip-shaped passages are gradually formed from the bottom to top with the seepage time. This is because the change rate of porosity is associated with the seepage velocity, which is larger in the area with bigger porosity. The change rate of porosity increases again with large seepage velocity. Finally the pores link together and become the leading passage of seepage.

![Porosity distribution images](image)

(a) the initial porosity  (b) porosity distribution after time of T=50000s

(c) porosity distribution after time of T=80000s  (d) porosity distribution after time of T=100000s

**Figure 5:** The change of porosity in Paleokarst collapse breccia pipe

Fig 6(a)–(d) describe the initial seepage velocity distribution and its changes after different erosion time. It could be seen that in beginning the velocity distribution is random and the seepage passages intersect each other to form network. The seepage velocity increases with time in which the maximum seepage velocity increases from $3.767 \times 10^{-4}$ m/s to $3.856 \times 10^{-3}$ m/s. Meanwhile, the number of major seepage channels gradually decrease. Their distribution makes a change from the network to the tape-shaped, which is similar with the porosity changes.
Figure 6: The change of velocity in Paleokarst collapse breccia pipe

Figure 7: Porosity curves in cross-section A→A′

Figure 8: Seepage velocity curves in cross-section A→A′
The porosity and seepage velocity changes in $\Lambda \rightarrow \Lambda'$ section of Paleokarst collapse breccia pipes are shown in Figure 7 and Figure 8., where we could see the final porosity and seepage velocity are both larger than initial. If the initial porosity and seepage velocity were smaller, the final changes of them could also be fewer. The reverse is applicable. This is because the permeability coefficient depends on the porosity. The larger porosity leads to the bigger seepage velocity and more intensive erosion, which induces the further increase of porosity.

![Figure 9: Fluid-particle velocity variation as the time at point B](image)

Figure 9: Fluid-particle velocity variation as the time at point B

Figure 9 is fluid and particle velocity variation as the time at point B, we can see that the vertical velocity of particles are smaller than that of fluid, and particle velocity as well as the different between particle and fluid increase with the grow of fluid velocity, this is because particles with larger velocity can induce bigger drag force, which can hinder particles transport to some degree.

![Figure 10: Particle concentration variation as the time at point B](image)

Figure 10: Particle concentration variation as the time at point B

Particle concentration variation as the time at point B is shown in Figure 10, the figure reveal that particle concentration first increase rapidly until it reach a peak value and then drop, this is because particles were eroded from Paleokarst collapse breccia pipe and transport under the fluid movement, which can cause the increase of particle concentration, and at the same time, as shown in Figure 11,
the porosity will also grow. With the increase of porosity, particles can be eroded will become smaller, and the particle concentration will drop.

![Porosity variation as the time at point B](image1)

**Figure 11:** Porosity variation as the time at point B

![Water flow rate from Paleokarst collapse breccia pipe with the time](image2)

**Figure 12:** Water flow rate from Paleokarst collapse breccia pipe with the time

Figure 12 shows the water flow rate changes against time in the model top boundary. The accelerated increase of flow could be observed and finally the water inrush is induced.

**CONCLUSIONS**

This paper mainly studied the seepage characteristics of Paleokarst collapse breccia pipes and water inrush mechanism induced by porosity development by means of building a particle erosion model. We summarized the main conclusions as following:

(1) When mining activity discovered Paleokarst collapse breccia pipes with aquifer at the bottom, it will be eroded constantly because of seepage and the porosity and penetration gradually increase. The seepage velocity also become larger and the water inrush of floor Paleokarst collapse breccia pipes would take place. In order to prevent it, some measures should be taken to reduce the erosion of filler in Paleokarst collapse breccia pipes such as grouting.
The permeability of Paleokarst collapse breccia pipes increases slowly in beginning. However there would be an accelerated increase in the seepage velocity with the increasing and connection of porosity. So if the Paleokarst collapse breccia pipes are uncovered, the water inrush could be prevented by some timely measures.

The theoretical analysis and simulation results in this paper should be applied to practical engineering. The solid-fluid coupling model for Paleokarst collapse breccia pipes should be further improved to reveal the water inrush mechanism.

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